Response to: Dave Chapman's "Comment on "Cross-shelf eddy heat transport in a wind-free coastal ocean undergoing winter time cooling""

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Chapman [2002] (hereafter C02) comments on the relation between the work of *Chapman* [1999] and *Chapman and Gawarkiewicz* [1997] (hereafter CG), and the results in *Pringle* [2001] (hereafter P01) in the limit of a cooling region confined to near the coast. The comment clarifies the relation between P01 and preceding work.

The main result of CG, reiterated in C02, is that the surface buoyancy flux into a polynya is quickly balanced by a horizontal cross-shelf buoyancy flux driven by a vigorous eddy field. This result of CG motivated the work of P01 and has remained a reliable guide to the evolution of the system throughout.

CG finds a timescale, t_e, after which eddies substantially modify the buoyancy budget of a coastal polynya. C02 points out that this timescale should remain valid even in the presence of bottom friction. C02 argues that because the polynyas are narrower then the length scales of the eddies which drive the cross-shelf buoyancy flux, the eddies will be able to remove buoyancy from the cooling region efficiently even if bottom friction subsequently retards the cross-shelf motion of the eddies. This is confirmed by all the model runs in P01 in which cooling was confined to near the coast, and should have been stated in P01.

C02's section 3 also provides an alternative derivation for the cross-shelf eddy driven buoyancy flux in the presence of friction. This derivation clearly shows why the flux laws of CG and P01 differ and how they are related.

1. "Fundamental changes" and their significance

C02 quotes P01 as saying "adding realistic levels of bottom friction fundamentally changes the results of CG," and then goes on to say "such strong criticism casts serious doubt on the reliability and usefulness of CG and related studies." I would hope the latter statement is not true (it was certainly not my intent for it to be so). To see why, it is useful to briefly review the "fundamental changes" describe in P01 and relate them to CG. There are two differences worth discussing -

the first is the form of the cross-shelf buoyancy flux law, the second is the nature of the longtime density evolution near the coast.

In P01, the form of the cross-shelf buoyancy flux law is found to be modified by bottom friction at longtime so that it depends on the magnitude of friction or the bottom slope (e.g. figures 7 and 9 of P01). The flux law in the frictional ocean at longtime becomes a function of the local cross-shelf density gradient, and not on the average density in the cooling region as in CG (see C02 section 3 or P02). Neither the dependence on friction nor density gradient exists in the flux law in CG – but this in no way reduces the significance or usefulness of the conclusion of CG that cross-shelf eddy fluxes dominate the buoyancy balance of a coastal polynya on timescales longer then t_e, and that the rate of increase of density in the polyna is dramatically slowed at t_e. All that is changed is the nature of the flux law. (The “longtime” referred to above is the time it takes the initial eddies to interact, cascade to larger scales, and reach an equilibrium lengthscale. References in P01 suggest that this time scales as an advective time scale formed from the eddy length scale and the geostrophic velocity, but this was not investigated in P01, where the emphasis was on the longtime behavior. In the model runs, the time scale seemed to be between 10 and 16 days.)

The change in the flux law changes the longtime behavior of the near shore density. In CG, and in earlier work it refers back to [*Visbeck et al.*, 1996], the cross-shelf buoyancy flux is a function of the mean density in the cooling region. This mean density was found to reach a steady state when it drove a cross-shelf buoyancy flux which balanced surface cooling. CG successfully scaled for this density. When experiments were run with bottom friction and a polynya of infinite alongshore extent, the rate of increase of density in the cooling region was found to slow dramatically, but not halt, at time t_e. It is reassuring to find quantitatively very similar behavior in the frictional runs of both P01 and C02 (figure 4 of P01, figure 2 of C02). But even as the density in the cooling region continued to increase in the frictional runs, the cross-shore buoyancy flux limits toward that necessary to balance the surface buoyancy flux.

C02 states that because “the offshore flux eventually achieves comparable magnitudes with or without bottom friction... bottom friction has not fundamentally changed the behavior.” It is worth amplifying on the last paragraph of P01’s section 3 to show why the cross-shelf buoyancy flux reaches the limit it does even while the density keeps growing, to demonstrate why these two behaviors are not contradictory, and thus to clarify the relation between P01 and C02 and to show what is changed and what is not changed by the addition of friction. (A more complete analysis of time variation of density is included in *Pringle* [1998].)

As described in section 3 of C02 and section 4.2.4 of P01, at longtime and in the presence of bottom friction, the cross-shelf buoyancy flux scales as some power of the local cross-shelf density gradient. Suppose that the majority of the coastal ocean affected by the cooling has achieved a balance between the net surface cooling and the offshore buoyancy flux, and that the magnitude of the cross-shelf density gradient is given by equation (17) of P01 or equation (13) of C02. This cross-shelf density gradient will be called $\overline{\rho_y}$ here. (P01, CG and C02 all use a somewhat confusing convention that y is the offshore coordinate.) An evolution toward this state in the flat bottom base case of P01 can be seen in the alongshore and depth average density in figure 1. With this assumption, the cross-shelf distribution of the density anomaly will be

$$\rho = \rho^{cool} - \overline{\rho_y} y \tag{1}$$

where ρ^{cool} is the density anomaly at the coast, under the assumption that the cooling region is narrow compared to the region affected by the cooling. The cross-shelf and depth integrated density anomaly is then $H(\rho^{cool})^2/(2\overline{\rho_y})$. ($H$ is the water depth - all notation that follows is that of C02.) Because of the surface buoyancy flux, this integrated density anomaly must increase linearly with time, as $\rho_0 g^{-1} B_0 bt$, where b is the cross-shelf extent of the cooling region, B_0 is the surface buoyancy flux in the cooling region, and t is the time since the onset of cooling.

Equating these two expressions for the depth averaged density leads to

$$\rho^{cool} = \sqrt{\frac{2\overline{\rho_y}}{H}\frac{\rho_0 B_0}{g} bt}. \tag{2}$$

Thus the density at the coast increases as $t^{\frac{1}{2}}$. A similar argument can be made for any point offshore at long enough time. The average density anomaly inshore of any point is proportional to the time integral of the difference between the horizontal flux needed to balance the entire surface buoyancy flux and the actual horizontal flux. In this case, the average density anomaly, and thus the time integral of the cross-shelf flux difference, would scale as $t^{\frac{1}{2}}$, and thus the difference between the actual flux and the buoyancy flux needed to balance cooling would scales as $t^{-\frac{1}{2}}$. It is thus consistent for the buoyancy flux to limit as $t^{-\frac{1}{2}}$ to the cross-shelf flux needed to balance the surface buoyancy flux even as the density near the shore grows without limit as $t^{\frac{1}{2}}$. The change in the buoyancy flux law described in P01, and its prediction of a nearshore density that grows without limit, is in no way contradictory to the predictions of CG and C02 or the numerical experiments which show the cross-shelf buoyancy fluxes growing to balance the surface buoyancy flux.

It is worth noting that (2) allows us to relate the rate of growth of density in the cooling region to parameters such as bottom friction at longtime. As described in P01, in the flat bottom case $\overline{\rho_y}$ scales as the bottom friction r to the one-third power, so that the density in the cooling regions is proportional to $t^{\frac{1}{2}}r^{\frac{1}{6}}$. This dependence on friction is weak, but is evident in a series of model runs from *Pringle* [1998] in which r is varied by nearly an order of magnitude (figure 2). The constants used in the scaling for $\overline{\rho_y}$ are those of P01, which are equivalent to $\gamma = 0.47$ in C02. The results are not inconsistent with those shown in C02. Figure 2 exhibits the dependency on friction more clearly, however, due to the longer model integrations and the slightly smoother density curves obtained by averaging over four smaller model runs initialized with very slightly different initial conditions (see P01 and *Pringle* [1998]).

2. Numerical issues

There is a difference in the frictionless numerical model runs of P01 and C02 (P01 figure 4 and C02 figure 2). In P01, the density increases by about 0.1 $\mathrm{kg\,m^{-3}}$ in the 33 days following t_e. This was interpreted as being nearly constant, in agreement with the prior literature. C02 finds that, under similar circumstances, the density increases by about 0.15 $\mathrm{kg\,m^{-3}}$. There are two differences in the model parameters which could perhaps explain these differences. In P01, the horizontal diffusivity of velocity was 7 $\mathrm{m^2\,s^{-1}}$, while in C02 it was 20 $\mathrm{m^2\,s^{-1}}$. For the ≈25 km eddies in the model runs, this implies diffusive timescales of about 1000 days and 360 days, respectively. It is hard to see how this would effect the results. The vertical mixing of density and momentum also differed, most significantly where the water was unstably stratified due to surface fluxes. P01 used a parameterization which matched observations of buoyancy forced convection in the atmosphere and agreed with Mellor Yamada in the limit of purely buoyancy driven convection [*Pringle*, 1998]. For the buoyancy fluxes in the flat bottom base case runs discussed here, the mixing coefficient in unstably stratified water is about 0.4 $\mathrm{m^2\,s^{-1}}$. C02 used a Mellor-Yamada scheme, but limits the mixing to less than $10^{-3}\,\mathrm{m^2\,s^{-1}}$ by fiat. The significance of this difference is unclear, but it seems likely to be relatively unimportant, for even in the cooling regions eddies efficiently restratify the ocean except within a few gridpoints of the wall. The origin of the difference in evolution of the frictionless cases remains unclear. The model code used in P01 is available upon request.

3. Conclusion

Nothing in P01 was meant to deny the importance of CG – it was, in fact, inspired by CG. Eddies dominate the flux of buoyancy across the boundary of a polynya any time more then t_e after the onset of cooling. P01 pointed out that the nature of the evolution of the mass field would be altered by the presence of friction and bottom slope, even as eddy fluxes remained of primary importance.

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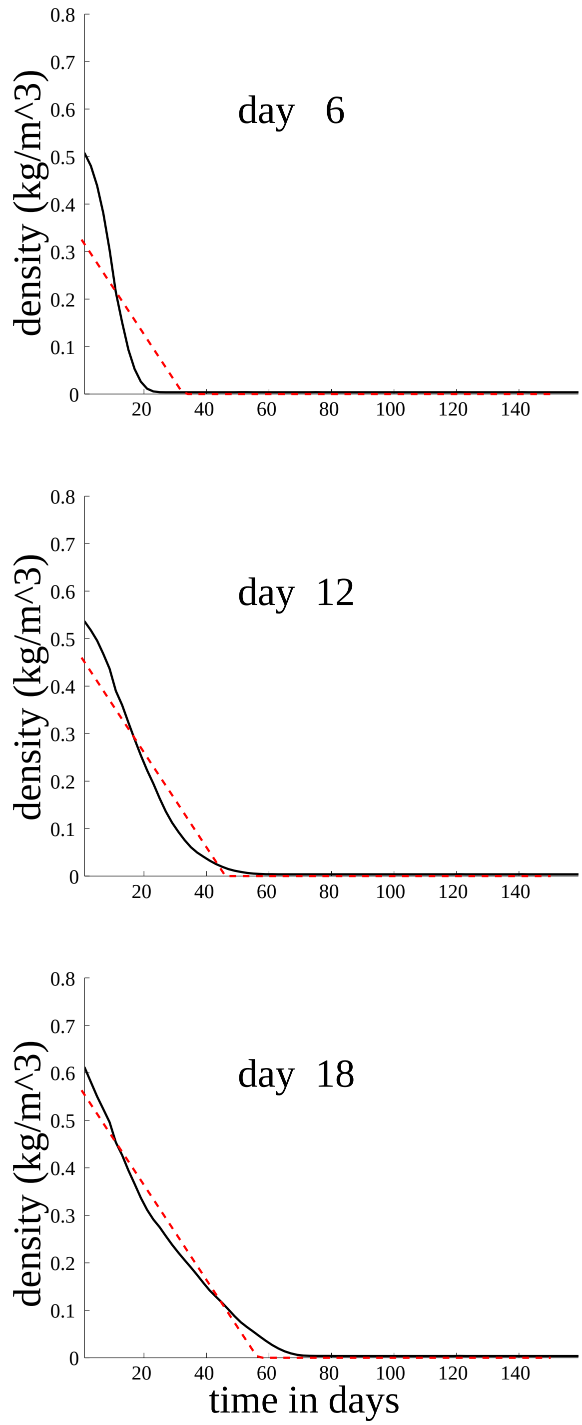

Figure 1: Solid line: The depth and alongshelf averaged density from the base flat bottom case of PO1. (P01's figure 4). Dashed line: the density predicted by (1), in which the cross-shelf density gradient is everywhere $\overline{\rho_y}$.

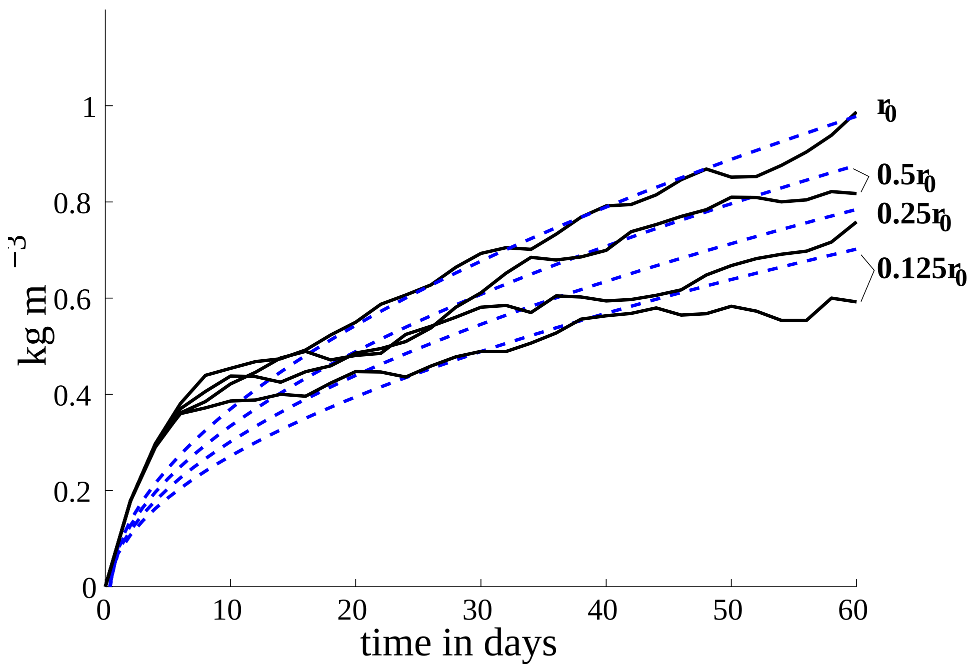

Figure 2: Solid line: The average density beneath the cooling region for four values of friction, from the the base flat-bottom run of P01 with a bottom friction of $r_0 = 4.5 \times 10^{-4}\,\mathrm{m\,s^{-1}}$, to runs with $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$th the bottom friction. The results for each value of friction is the average of four model runs initialized with randomly slightly differing initial conditions, as described in P01.

Dashed lines: The predicted density evolution from (2) for all four values of bottom friction, reduced by $\frac{1}{2}\overline{\rho_y}b$ to account for a finite width cooling region.