

# Enhancement of wind-driven upwelling and downwelling by alongshore bathymetric variability

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## ABSTRACT

Steady wind-driven flow along a shelf of changing width is described with a frictional barotropic model valid in the limit of small Rossby and Burger number. In these limits, an alongshore wind drives enhanced on-shelf transport in a coastal ocean if the shelf widens downwind, and the change in shelf width only affects the flow in the direction of Kelvin wave propagation (“down-wave”) from the change in shelf width. There is enhanced onshore transport of cold, nutrient laden bottom water if the winds are upwelling favorable and the shelf narrows in the direction of Kelvin wave propagation. This enhanced transport extends a considerable distance away from the change in shelf width, but becomes concentrated near the shelf break far from the change in width, as in *Chapman* [1986]. Isobath curvature on the scale of the shelf width significantly modifies local cross-shelf transport. The cross-shelf transport of nutrient rich water during upwelling is expected to be enhanced from Pt. Eugenia to La Jolla, San Luis Obispo to Monterey, and Pt. Reyes to Cape Mendocino on the west coast of North America.

# 1. Introduction

If a wind blows along a shelf in the direction that the shelf widens, more water will be forced onto the shelf from the deep ocean than if the shelf were of uniform width. Why this must be can be seen from the depth-averaged shallow-water alongshore-momentum equation in water of depth  $H$  and in the limit of no alongshore variation:

$$f\bar{u}^0 = -g\bar{\eta}_y^0 + \frac{\tau_{\text{top}}^y}{H\rho_0} - \frac{r}{H}v. \quad (1)$$

$\bar{u}$ , the depth-averaged cross-shelf velocity, is zero because of the coastal boundary condition, and  $\bar{\eta}_y$ , the alongshore surface pressure gradient, must be zero by the assumption of no alongshore variation. This leaves a balance between the bottom stress (here parameterized as a linear drag  $r$  times the the depth-averaged alongshore velocity  $v$ ) and the surface wind stress  $\tau_{\text{top}}^y$ . The alongshore velocity is then

$$v_{2D} = \frac{\tau_{\text{top}}^y}{\rho_0 r}. \quad (2)$$

This velocity is everywhere uniform and downwind. Now consider a shelf which abruptly narrows. Consider two sections across the shelf far enough away from the change in shelf width that the flow through each section has adjusted to the new shelf width, has ceased to vary alongshore, and obeys (2). Since the velocity through the two sections will be the same, there must be greater transport through the wider section, for its cross-sectional area is greater. Thus if the wind is blowing, and the current flowing, in the direction of a widening shelf, there must be a net transport of water from the deep waters onto the shelf to provide this additional transport.

This onshelf transport can be considerable. For a wind-stress of  $10^{-1}$  Pascals (or about a  $10 \text{ m s}^{-1}$  wind) and a bottom drag coefficient of  $r = 5 \times 10^{-4} \text{ m s}^{-1}$ , (2) predicts an alongshore velocity of about  $20 \text{ cm s}^{-1}$  for regions well away from the change in shelf width. If a coastal ocean has a shelf-break depth of 100m, and if the width of the shelf goes from 15 to 30km, the cross-sectional area of the shelf will change by about  $7.5 \times 10^5 \text{ m}^2$ . This change in shelf width,

multiplied by the alongshore velocity of  $20 \text{ cm s}^{-1}$ , implies that  $0.15 \text{ Sv}$  of water must enter the shelf break in the vicinity of the change in shelf width. The importance of these cross-shelf flows at a given point on the shelf will be governed by the along-shelf distance over which the additional transport enters or leaves the shelf, and its importance to the biology and chemistry of the coastal ocean will be governed by the vertical distribution of the additional cross-shelf transport.

In order to discover where the flow is modified by the change in shelf width, it is necessary to create a simplified model of coastal wind driven flows. Careful analysis of the alongshore momentum balances in some wind-driven coastal oceans have found that the dominant balance was between windstress, bottom drag, alongshore pressure gradients, and acceleration (*Lentz and Winant* [1986] near San Diego in winter, *Lentz* [1987] in the Coastal Ocean Dynamics Experiment (CODE) region in spring, *Lee et al.* [1989] in the South Atlantic Bight, and *Lentz et al.* [1999] and *Lentz* [2001] off North Carolina.) The simplest model which captures these dynamics is a linear homogeneous model with bottom friction. To further simplify the problem, time variation will be ignored, which is appropriate for timescales longer than the frictional spin-down time of the shelf [*Dever*, 1997]. The papers above find that linear bottom drag coefficients of about  $r = 2 \times 10^{-4}$  to  $5 \times 10^{-4}$  agree well with observations, which suggests a frictional spindown time  $H/r$  of about 1 to 3 days in 50 meters of water (c.f. *Dever* [1997]).

Because the model below is steady, homogeneous and frictional, it will not apply when the cross-shelf advection of buoyancy causes arrest in the bottom boundary layer, eliminating friction (e.g. *Garrett et al.* [1993] or *Trowbridge and Lentz* [1998]) or when the alongshore buoyancy gradients are important in forcing cross- or along-shelf flows (*Lentz and Trowbridge* [2001] in the winter CODE region, *Lentz and Winant* [1986] in San Diego in summer, or *Austin* [1998] a frictional spindown time after the cessation of wind).

In the next two sections, a modified version of *Csanady's* [1978] arrested topographic wave equation is derived which includes the effects of a changing alongshore bathymetry, and the changes in upwelling and downwelling along a shelf of changing width are described. It will

be argued that shelves that narrow in the direction of Kelvin wave propagation will be sites of enhanced onshelf transport near the bottom during upwelling, and thus enhanced onshelf fluxes of nutrients. A short comparison of the theory to some data will be made, and predictions of observable consequences of the theory will be given.

## 2. Model Equations

The neglect of acceleration, non-linearity, and stratification is only valid in several limits (see *Hogg [1980]*). *Janowitz and Pietrafesa [1982]* show by a formal expansion in small parameters that stratification and the non-linear advection of relative vorticity can be neglected relative to the advection of the planetary potential vorticity  $f/H$  when

$$\left(\frac{L_D}{L}\right)^2 \leq \epsilon \ll \frac{\Delta H}{H}, \quad (3)$$

where  $L_D$  is the internal radius of deformation,  $L$  is the lengthscale of the alongshore variation of the bathymetry,  $\Delta H$  the variation of the water depth that a streamline could encounter,  $H$  a scale water depth, and  $\epsilon$  is the Rossby number,  $V(fL)^{-1}$ .  $V$  is a velocity scale for the geostrophic interior and  $f$  is the Coriolis parameter (*c.f. Allen [2000]*).

Inertia can be neglected when it is small not only relative to the geostrophic balance, but also to friction. This is true when  $VT L_y^{-1} \ll 1$  for a flow whose speed scales as  $V$ , varies over an alongshore length scale  $L_y$ , and is slowed with a frictional timescale  $T$ . The results below are self-consistent with this limit, for it will be found below that the length scale of alongshore variation of alongshore velocity is of the order of hundreds of kilometers, and reasonable values for the other parameters are  $V = 0.2 \text{ m s}^{-1}$ , and 2 to 4 days for  $T$  [*Dever, 1997; Lee et al., 1989; Lentz et al., 1999*], so the ratio of inertial to frictional forces is of order  $O(10^{-1})$ . On time scales larger than  $T$ , the flow can be treated as quasi-steady [*Dever, 1997*].

The steady, linear, homogeneous equations of motion, valid in the limits described above, are:

$$-fV = -gH\eta_x + \frac{\tau_{\text{top}}^x}{\rho_0} - \frac{\tau_{\text{bot}}^x}{\rho_0} \quad (4a)$$

$$fU = -gH\eta_y + \frac{\tau_{\text{top}}^y}{\rho_0} - \frac{\tau_{\text{bot}}^y}{\rho_0} \quad (4b)$$

$$U_x + V_y = 0 \quad (4c)$$

where  $U$  and  $V$  are the depth integrated cross- and alongshore velocity. To close these equations, the bottom stress  $\tau_{\text{bot}}$  must be related to the depth integrated flows  $U$  and  $V$ . It is convenient, and realistic in the presence of strong tidal and surface wave driven flows, to assume a linear drag law:

$$\tau_{\text{bot}} = \rho_0 r \mathbf{u}_{\text{bot}} \quad (5a)$$

$$r \approx 2C_d \langle \mathbf{u}_{\text{bot}} \rangle \quad (5b)$$

where  $\langle \mathbf{u}_{\text{bot}} \rangle$  is the root-mean-square of all bottom velocities, not just the low frequency ones of interest here [Lentz *et al.*, 1999; Wright and Thompson, 1983]. The near bottom velocity must then be related to the depth integrated velocity, and here is set to the depth integrated velocity divided by the water depth:

$$\tau_{\text{bot}} = \rho_0 r \frac{\mathbf{U}}{H}. \quad (6)$$

This can be shown to be reasonable for the surface pressure gradient forced flows in weakly stratified oceans [Brink and Allen, 1978]. It is more problematic for surface Ekman flows, which are trapped to the surface by rotation and thus should not feel bottom drag. The error in the surface Ekman transport scales as  $r^2 H^{-2} f^{-2}$ , and so this analysis will be confined to where that parameter is small.

The transport streamfunction  $U = -\Psi_y$  and  $V = \Psi_x$  can be used to re-write (4) as a potential vorticity equation:

$$0 = \underbrace{J\left(\Psi, \frac{f}{H}\right)}_{\text{Advection of } \frac{f}{H}} + \underbrace{\nabla \cdot \left(\frac{r}{H^2} \nabla \Psi\right)}_{\text{Friction}} - \underbrace{\nabla \times \left(\frac{\tau_{\text{top}}}{\rho_0 H}\right)}_{\text{Wind Torque}}. \quad (7)$$

There are only three terms in linear, steady and homogeneous potential vorticity equation (7) – the advection of planetary potential vorticity, the dissipation of relative vorticity by bottom friction and other frictional effects, and the input of potential vorticity by a curl in the windstress.

The latitudinal variation in  $f$  will be neglected below, because for the alongshore length scales described below the cross-shelf topographic gradient of potential vorticity is much more important than the planetary gradient of potential vorticity.

### 3. Changing Shelf Width and Wind-Driven Flows

Equation (7) can be solved numerically (see appendix), and a solution is shown in figure 1 for a uniform upwelling wind along a northern-hemisphere shelf whose width halves from 30 km in the south to 15 km to the north and for a uniform downwelling wind along a shelf whose width doubles to the north. The shelf slopes linearly from 10 meters at the coast to 110 meters at the shelf break, then deepens to 3000 meters over the next 40 km offshore. The coastal boundary condition is no flow through the coast, while the other boundary conditions, detailed in the appendix, are chosen so the solution is nearly identical to the solution for an  $f$ -plane ocean which extends infinitely and without change to the north, south and west. In all model runs presented below,  $f = 10^{-4} \text{ s}^{-1}$  and  $r = 5 \times 10^{-4} \text{ m s}^{-1}$ . The cross-shelf grid spacing is 0.65 km, the along-shelf grid spacing is 1 km, and the problem is solved on a 385 by 385 grid (only part of which is shown in the figures). The solution changes by less than one percent when the resolution is doubled, and by less than ten percent when it is halved.

The depth averaged currents in panel A of figure 1 follow the isobaths where the shelf changes width – thus the alongshelf flow is nearly twice as fast in the narrow region near the change in shelf width. Panel B shows the strength of the alongshore flow, scaled by the strength it would have if there were no variation in the shelf width (from (2)). It is apparent that the flow immediately to the south of the change in shelf width is not affected by the change in shelf width,

and all adjustment to the change in shelf width occurs to the north of the change in both figures. The flow adjusts first near the coast, and the adjustment moves progressively offshore farther to the north. Panel C shows the net cross-shelf transport at the midpoint of the shelf. Consistent with the first two panels, there is no net cross-shelf transport south of the change in shelf width – the flow is two dimensional, and the surface Ekman flow is balanced by a bottom Ekman flow. North of the change in shelf width, there is a net onshore transport of water, in both figures, because in both the shelf widens in the direction of the alongshore flow.

This raises four questions. Why does the change in shelf width only affect the flow to the north of the change? Why does the adjustment to the new shelf width occur first near the coast? How far to the north of the change in shelf width is it necessary to go before the flow is not affected by the change? And where does the additional flow enter the shelf, in the bottom boundary layer, geostrophic interior, or surface Ekman layer?

To answer these questions, it is useful to write the vorticity equation (7) in a coordinate system aligned to the isobaths. This coordinate system is illustrated in figure 2. The coordinate  $n$  is normal to isobaths, and takes the place of  $x$  when isobaths parallel the coast, and  $p$  is parallel to an isobath and takes the place of  $y$  when isobaths parallel the coast. At any point the coordinate system  $(\hat{n}, \hat{p})$  can be chosen so it is only a rotation from  $(\hat{x}, \hat{y})$ . Equation (7) is then locally

$$\frac{\partial \psi}{\partial p} \frac{\partial}{\partial n} \left( \frac{f}{H} \right) = \frac{\partial}{\partial n} \left( \frac{r}{H^2} \frac{\partial \psi}{\partial n} \right) + \frac{1}{R} \left( \frac{r}{H^2} \frac{\partial \psi}{\partial n} \right) + \frac{\partial}{\partial p} \left( \frac{r}{H^2} \frac{\partial \psi}{\partial p} \right) - \nabla \times \left( \frac{\tau_{\text{top}}}{\rho_0 H} \right), \quad (8)$$

where  $R$  is the radius of curvature of the isobath at that point.

Equation (8) can be scaled, and a consistent balance found, if the along-isobath length scale is assumed to be  $Hf/r$  times the across-isobath length scale. The third term on the righthand side of (8) is found to scale as  $r^2 H^{-2} f^{-2}$  times the lefthand side and the first term on the righthand side. Since for typical continental shelves,  $r^2 H^{-2} f^{-2}$  is small, the third term on the right-hand side is dropped (for  $H = 50\text{m}$ ,  $r = 5 \times 10^{-4} \text{ m s}^{-1}$ , and  $f = 10^{-4} \text{ s}^{-1}$ ,  $r^2 H^{-2} f^{-2}$  is  $10^{-2}$ ). This



leaves

$$\frac{\partial \psi}{\partial p} = \left[ \frac{\partial}{\partial n} \left( \frac{f}{H} \right) \right]^{-1} \left[ \frac{\partial}{\partial n} \left( \frac{r}{H^2} \frac{\partial \psi}{\partial n} \right) + \frac{1}{R} \left( \frac{r}{H^2} \frac{\partial \psi}{\partial n} \right) - \nabla \times \left( \frac{\tau_{\text{top}}}{\rho_0 H} \right) \right]. \quad (9)$$

This is a modified version of Csanady’s arrested topographic wave equation (with forcing) written locally in an isobath following coordinate system [Csanady, 1978].

Csanady pointed out the similarity of this equation to a heat equation, with the alongshelf direction taking the place of time. It can be solved as an “initial value problem” by integrating  $\psi$  not in time but along an isobath in the direction of long coastal trapped waves (i.e. with greater  $f/H$  on the right). Integrating in the opposite direction would lead to spontaneous singularities from arbitrary initial conditions just as integrating the heat equation backwards in time would. Thus any change in the forcing, bathymetry, et cetera only alters the solution to (9) in the direction a long coastal trapped wave will propagate. Thus:

Conclusion 1: Any change in shelf width will only affect the flow in the direction of long coastal trapped wave propagation (poleward on a west coast, equatorward on an east coast).

For convenience, the direction in which long coastal trapped waves propagate will be referred to as the “downwave” direction.

Downwave of the change in shelf width the flow will adjust to the new shelf width. As described in Csanady [1978], the solution will adjust first near the coast, and the influence of the coastal boundary condition will “diffuse” into the interior. The offshore extent of the adjusted region can be estimated by scaling (9) downwave of the region of changing shelf width. Re-writing (9) by expanding the first term on the righthand side and dropping the second term (because  $R = \infty$  along straight isobaths) leads to an equation which scales as

$$\begin{aligned} \frac{\partial \Psi}{\partial p} &= \frac{2r}{Hf} \frac{\partial \Psi}{\partial n} - \frac{r}{f} \left( \frac{\partial H}{\partial n} \right)^{-1} \frac{\partial^2 \Psi}{\partial n^2} + \text{forcing} \\ \frac{\Psi}{L^y} &= \frac{2r}{Hf} \frac{\Psi}{L^x} - \frac{r}{f} \left( \frac{\partial H}{\partial n} \right)^{-1} \frac{\Psi}{L^{x^2}} + \text{forcing} \end{aligned} \quad (10)$$

where  $L^y$  is an alongshore lengthscale and  $L^x$  is the distance offshore. If the depth is assumed to increase linearly away from the coast, so  $H = \alpha x$  and  $\partial H/\partial n = -H/L^x$ , the two terms on the right-hand side are of the same magnitude. All the terms of (10) will be of the same magnitude at a distance  $L^x$  offshore when  $L^y$  is equal to

$$L^{\text{fric}} = \frac{1}{2} \frac{Hf}{r} L^x = -\frac{1}{2} \frac{f}{r} H^2 \left( \frac{\partial H}{\partial x} \right)^{-1}$$

(the factor of 1/2 is a scaling factor which arises from a comparison of  $L^{\text{fric}}$  to the numerical solutions of (7)). When much less than  $L^{\text{fric}}$  from the change in shelf width ( $L^y \ll L^{\text{fric}}$ ), the term on the right-hand side of (10) are negligible,  $\partial\psi/\partial p \approx 0$ , and the streamfunction is constant on a line of constant  $f/H$ . When much farther than  $L^{\text{fric}}$  downwave from the change in shelf width, the terms on the righthand side of (10) dominate, there is a balance between the frictional terms and the forcing, and the flow has equilibrated to the new shelf width. Thus:

Conclusion 2: A distance  $L^x$  offshore, at an isobath of depth  $H$ ,  $L^{\text{fric}}$  is the distance downwave from the change in shelf width to the place where the flow has adjusted to the new shelf width.  $L^{\text{fric}}$  increases offshore.

Less than  $L^{\text{fric}}$  from the change in shelf width,  $\psi$  is constant on a line of constant  $f/H$ . Thus where the shelf narrows, streamlines converge and the flow is faster, and where the shelf widens the flow is slower. Since the flow upwave of the change in shelf width is unaffected by the change in width, the along-isobath velocity downwave of the change in shelf width will be approximately the ratio of the upwave isobath spacing to the downwave isobath spacing times the alongshore velocity which would exist in the absence of a change in shelf width (2) (*c.f.* figures 1 and 3).

If the shelf narrows downwave there is excess transport on the downwave shelf, relative to the two-dimensional solution. As the solution first adjusts to the new shelf width near the shore, the excess transport moves offshore. This causes an offshore jet to form downwave of the change in shelf width. Farther downwave from the change in shelf width, more of the shelf will have adjusted to the new width and the jet must move farther offshore. The inner edge of the jet is

roughly marked by the locus of points where  $L^{\text{fric}}$  is equal to the distance from the change in shelf width (figure 3). *Chapman* [1986] has studied this jet in the more general and realistic case where the bottom friction  $r$  decreases offshore. The depth dependent  $r$  narrows and elongates the jet, and traps it more strongly to the shelf break. An identical argument can be made for the elongated region of anomalously low flow, an “anti-jet,” downwave of a widening shelf (figure 1).

In order to understand how the change in shelf width effects the biology, chemistry and heat content of the coastal ocean, it is necessary to understand where additional transport caused by the change in shelf width enters the coastal ocean. Is it in the warm, nutrient poor surface waters, the entire water column, or in the cold, nutrient rich waters transported by the bottom Ekman layers?

The strength of the cross-isobath surface Ekman, geostrophic, and bottom Ekman transport can be found from (9), as re-written below:

$$-\frac{\partial\psi}{\partial p} = \left[ \frac{\partial}{\partial n} \left( \frac{f}{H} \right) \right]^{-1} \left[ -\frac{r}{H} \frac{\partial}{\partial n} \left( \frac{1}{H} \frac{\partial\psi}{\partial n} \right) + \frac{r}{H^2} \frac{\partial H}{\partial n} \left( \frac{1}{H} \frac{\partial\psi}{\partial n} \right) - \frac{1}{R} \frac{r}{H} \left( \frac{1}{H} \frac{\partial\psi}{\partial n} \right) + \nabla \times \left( \frac{\tau_{\text{top}}}{\rho_0 H} \right) \right] \quad (11)$$

where  $-\partial\psi/\partial p$  is the cross-isobath transport and  $H^{-1}\partial\psi/\partial n$  is the along-isobath velocity.

The first term on the right-hand-side is the cross-isobath transport caused by the frictional removal of the relative vorticity associated with the cross-isobath gradient of along-isobath velocity – *i.e.* Ekman Pumping. Where the relative vorticity is positive, positive potential vorticity is removed by friction. Since in this linear model, the potential vorticity is  $f/H$  and it must become less,  $H$  must become greater and there is an offshore cross-isobath transport (for  $f > 0$ ). The opposite occurs if the flow has negative relative vorticity – negative vorticity is dissipated, and there is an onshore cross-isobath transport. Thus cyclonic relative vorticity leads to offshore transport and anti-cyclonic relative vorticity to onshore transport. This mechanism moves the entire water column across-isobaths – the cross-isobath transport is geostrophic and associated

with along-isobath pressure gradients.

The third term on the right-hand side represents the same physical process, but the relative vorticity is associated with flow along curving isobaths, not cross-isobath gradients of the along-isobath velocity. As above, if the flow along isobaths curves cyclonically, there is an offshore transport, if anti-cyclonically, an onshore transport. The cross-isobath transport is geostrophic, extends through the water column, and is proportional to the inverse of the isobath curvature.

The second term on the right-hand side does not directly involve vorticity dynamics, but instead describes the bottom Ekman transport. Direct substitution of a uniform along-isobath transport  $V_g$  into this term produces, in the limit of  $r^2 H^{-2} f^{-2}$  small, a cross-shelf transport of  $-rH^{-1}f^{-1}V_g$ , which is, from (6),  $-\rho_0^{-1}f^{-1}\tau_{\text{bot}}^p$  – the bottom Ekman transport. This transport does not depend on the interior relative vorticity, and will be confined to within an Ekman depth of the bottom.

The fourth term on the right hand side of (11) forces the surface Ekman transport when there is no curl in the wind-stress. Substituting a uniform along-isobath wind into the fourth term on the right-hand side of (11) produces cross-isobath transports equal to  $\rho_0^{-1}f^{-1}\tau_{\text{top}}^y$ , the surface Ekman transport, in the limit of  $r^2 H^{-2} f^{-2}$  small. This transport will be trapped within an Ekman depth of the surface.

The relative strength of each term on the right-hand side of (11), and thus the relative strength of the surface Ekman, geostrophic, and bottom Ekman cross-isobath transports, can be estimated near the change in shelf width. Within  $L^{\text{fric}}$  of the change of shelf width, the streamlines nearly follow the isobaths, and the along-isobath velocity scales as the ratio of the upwave to downwave shelf width:

$$\text{along isobath velocity} = \frac{1}{H} \frac{\partial \psi}{\partial n} = v_{2D} \frac{1}{\alpha} \frac{\partial H}{\partial n} \quad (12)$$

where  $v_{2D}$  is the velocity upwave of the change in shelf width (from (2)), and  $\alpha$  ( $< 0$ ) is the bottom slope upwave of the change of shelf width on the isobath  $H$ . In the results described above and shown in figures 1 through 3, the bottom slopes linearly offshore until the shelf break,

and so  $\alpha$  is not a function of depth. In these limits the equation for cross-isobath transport (11) reduces to

$$-\frac{\partial\psi}{\partial p} = v_{2D}H \left( \frac{r}{Hf} \right) \left[ \frac{H}{\alpha} \left( \frac{\partial H}{\partial n} \right)^{-1} \frac{\partial^2 H}{\partial n^2} - \frac{1}{\alpha} \frac{\partial H}{\partial n} + \frac{H}{\alpha} \frac{1}{R} \right] + \underbrace{\text{Surface Ekman}}_{\approx v_{2D}H \left( \frac{r}{Hf} \right)}. \quad (13)$$

The first term inside the square brackets of (13) controls the cross-shelf transport driven by the frictional removal of the relative vorticity associated with the cross-isobath gradient of along-isobath flow. The cross-isobath transport scales as the surface Ekman transport times the ratio of the shelf width ( $H\alpha^{-1}$ ) to the cross-shelf length scale of the variation in the alongshelf velocity ( $(\partial H/\partial n)/(\partial^2 H/\partial n^2)$  from (12)). It is small, since  $\partial^2 H/\partial n^2$  is small where the depth increases linearly offshore to the shelf break, as it does in figures 1 through 3. This flow would be geostrophic, and extend through the water column. It is the linear, frictional equivalent to the cross-isobath transport described by *Janowitz and Pietrafesa* [1982].

The second term inside the square brackets of (13) controls the bottom Ekman layer cross-isobath transport. It scales as  $rH^{-1}f^{-1}$  times the along-isobath velocity, and thus, from (12), as the inverse of the ratio of the change in shelf width times the surface Ekman transport. Where the shelf narrows downwave ( $\alpha^{-1}(\partial H/\partial n) > 1$ ), it becomes greater than the surface Ekman transport, where the shelf widens downwave, it becomes less than the surface Ekman transport. The change in the bottom Ekman transport dominates the change in cross-isobath transport downwave of where the shelf changes width (figures 4 and 5). The bottom Ekman transport is trapped to within an Ekman depth of the bottom.

The third term inside the square brackets of (13) controls the cross-shelf transport driven by the frictional removal of the relative vorticity associated with flow along curving isobaths. It scales as the surface Ekman transport times the ratio of the topographic lengthscale ( $H\alpha^{-1}$ , or approximately the distance offshore) to the radius of curvature of the isobaths,  $R$ . When the shelf width changes by  $O(1)$  in a distance comparable to the shelf width, as in figures 1 through 5, the radius of curvature will be approximately the shelf width where the shelf changes width, and will

be of one sign when the shelf starts changing width and the other sign when the isobaths straighten out. This curvature drives a cross-shelf transport comparable to the surface Ekman transport in the region of changing shelf width, onshore where the flow curves anti-cyclonically, offshore where it curves cyclonically (figures 4 and 5). This cross-isobath transport is geostrophic, and will extend through the water column. Since the curvature driven flow will tend to be of equal magnitude but opposite sign as the shelf starts to narrow and stops narrowing, the net curvature driven cross-isobath transport in the region of changing shelf width is small. Once the isobaths parallel the coast again, the curvature term is zero.

Thus a change in shelf width changes the cross-isobath transport primarily by changing the strength of the bottom Ekman transport. From this we can understand how the net onshore transport downwave of the two changes in shelf width in figure 1 differ in their implications for the biology, heat content, and chemistry of the coastal ocean. In the top panel of figure 1, an upwelling wind blows along a coast which narrows in the direction of long coastal trapped wave propagation. The alongshore flow is faster where the shelf narrows. This faster flow enhances the onshore flow in the bottom Ekman layer, so that it is stronger than the offshore surface Ekman transport. There is then a net onshore transport of cold, nutrient laden bottom water onto the shelf (figure 6A). Conversely, in the bottom panel of figure 1, a downwelling wind blows along a shelf which widens downwave, and the alongshore flow is slower downwave. This decelerated flow weakens the offshore flow in the bottom Ekman layer so that it is weaker than the onshore surface Ekman transport. Thus there is also a net transport of water onto the shelf, but it is warm, nutrient poor surface water (figure 6B). Thus the final, and perhaps most important conclusion from this section:

Conclusion 3: The transport of cold, deep, nutrient laden water onto the shelf is enhanced above what it would be along a straight shelf only during upwelling along a shelf which narrows downwave.

Of course, any enhancement of cross-shelf nutrient transport in the bottom boundary layer does

not automatically lead to new production on the shelf - some other mechanism must bring the nutrients into the euphotic zone.

#### 4. Present and Possible Comparisons to Data

Several of the conclusions above can either be tested against available data or against data that could feasibly be gathered. Below this will be done or described.

It is predicted above that the flow should adjust to the new shelf width a distance  $L^{\text{fric}}$  downwave of a change in shelf width. Because  $L^{\text{fric}}$  is less in shallower water, the adjustment occurs first near the coast. After adjustment, the surface and bottom stress are equal and surface and bottom Ekman transports are equal and opposite, and so that there is no net cross-shelf transport. It might seem practical to test these predictions by measuring where the surface and bottom stress is equal and the alongshore flow agrees with (2).

However, alongshore variation in the wind stress complicates the test for adjustment, for even along a perfectly straight and uniform shelf, the flow at a point is not driven by the winds at that point. *Winant* [1979], using a variation of (9), and *Allen and Denbo* [1984], using a coastal trapped wave formalism, show that the flow at a point is governed by the winds upwave of that point. In the steady, linear and weakly stratified limit described above, the coastal wind  $L^{\text{fric}}$  upwave govern the currents at an offshore point.

It is rare to have a good knowledge of the alongshore variation in the winds. It is, however, possible to study the adjustment of the coastal ocean on a shelf which suddenly widens downwave from nearly zero width. Since the downwave flow will be weakened by the ratio of upwave to downwave shelf width, if the upwave shelf has zero width, the upwave influence is infinitely weakened. The alongshore flow will be weak immediately downwave of the widening shelf, and will increase nearer to shore and farther downwave. *Schwing* [1989] examined such a shelf, the Scotian Shelf to the south-west of the Laurentian Channel. The Laurentian Channel truncates

the shelf upwave of Nova Scotia. Schwing computed the expected sea-surface pressure response along Nova Scotia with a version of (9) modified to include time variation, and compared it to the observed variation in sea surface pressure. In good agreement with the model in his paper and above, the amplitude of the local forced wave response of the ocean to the winds increased both downwave and nearer to the coast. He also found remotely forced sea level pressure variation to be important on the Scotian Shelf. A similar shelf exists near San Francisco, CA. At Monterey Bay, the shelf width is nearly zero, and increases to the north. The response of this shelf to an alongshore wind can be examined in a highly idealized model of the shelf north of Monterey, in which the coast is straight, and the shelf widens from nearly zero width to the north, remains a uniform width near San Francisco, and narrows between Pt. Reyes and Pt. Arena. The depth is 10 meters at the coast, and 150 meters at the shelf break. Offshore of the shelf-break the depth increases to 2150 meters within 100 km.<sup>1</sup> Figure 7 shows the transfer function between the wind and the along-isobath velocity, normalized by the value it would have along a shelf of uniform width. There is in the solution a shadow zone offshore of the isobath  $L^{\text{fric}}$  downwave of the initial widening of the shelf. These areas are weakly forced by the wind. Observations should show that the transfer function between winds and currents decreases markedly offshore between the coast and shelf-break in the region 20 to 200km north of Monterey. The currents in the shadow zone should be driven mainly by baroclinic effects and windstress curl. Once the shelf begins to narrow near Pt. Reyes, the alongshore currents increase.

It is also predicted above that the on-shelf transport of cold, nutrient rich water will be enhanced where there is upwelling along a coast which narrows in the direction of long coastal trapped wave propagation. The alongshelf transport will be significantly altered, and thus a significant amount of water added to the shelf, over a distance of  $L^{\text{fric}}$ , defined with the depth of the shelf break. For a shelf break depth of 150 m, bottom friction of  $r = 5 \times 10^{-4} \text{ m s}^{-1}$ , and shelf widths of 5 to 30 km,  $L^{\text{fric}}$  will range from 75 to 450 km. On figure 8, regions where the shelf

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<sup>1</sup>The shelf-break slope is reduced for numerical convenience. It has little effect on the solution.



from Baja California to Cape Mendocino narrows downwave for a hundred or more kilometers are marked.

From Pt. Eugenia to La Jolla, San Luis Obispo to Monterey, and the CODE region to Cape Mendocino, the shelf narrows to the north and thus downwave. Measurements in these regions, taken when there are upwelling winds, should confirm that there is more onshore transport in the bottom Ekman layer than offshore transport in the surface Ekman layer. These observations are hard to make – small pointing errors in the current meters can fold the strong along-isobath currents into the much weaker cross-isobath currents. It will likely be more robust to measure the alongshelf transport at either end of these regions. There ought to be  $O(1)$  changes in alongshore transport, with more transport at the southern, upwave ends of the shelves. The southern ends of these regions should also have lower mean heat contents and higher mean nutrient concentration, while heat and nutrient budgets should require fluxes of heat off the shelf, and nutrients onto the shelf, to close.

## 5. Conclusion and Discussion

The main conclusions of this work are four-fold. First, in the weakly stratified, linear and frictional limits described in section 2, the effects of any change in the shelf width are confined to the region of changing shelf width and downwave of the change. Thus on a western coast, a change in shelf width only affects the flow poleward of the change, and on east coasts equatorward.

Second, the effects of the change in shelf width are felt for a distance  $L^{\text{fric}}$  downwave of the change in shelf width.  $L^{\text{fric}}$  increases with depth. For a shelf typical of the western coast of North America, the bottom slope is about  $5 \times 10^{-3}$ ,  $f = 10^{-4} \text{ s}^{-1}$ , and  $r = 5 \times 10^{-4} \text{ m s}^{-1}$ , so  $L^{\text{fric}}$  is 8km on the 20 meter isobath, 50km on at 50 meters, and 200km on the 100 meter isobath. Thus the shallow waters adjust to new shelf widths most rapidly, returning to a balance between surface and bottom Ekman transports, while at greater depths the effect of a change in shelf width

is felt farther downwave.  $L^{\text{fric}}$  also increases as shelf width increases – thus on wider shelves, the effect of a change in shelf width is felt farther downwave.

Third, the transport between an isobath and the coast is conserved over distances much less than  $L^{\text{fric}}$ . This and the first result imply that where the shelf narrows downwave, the wind-driven flow is made stronger than it would be otherwise, and where the shelf widens downwave, the flow is made weaker. This and the second result imply that a jet, where the shelf narrows downwave, or anti-jet, where it widens, will be formed offshore and downwave of the change in shelf width. The inshore boundary of the jet or anti-jet is  $L^{\text{fric}}$  downwave from the change in shelf width. *Chapman* [1986] has studied similar jets, and found that they are concentrated near the shelf break and elongated when more realistic bottom friction schemes, in which  $r$  decreases offshore, are used. This is because smaller bottom friction causes larger  $L^{\text{fric}}$  offshore.

The fourth major conclusion is that there will be enhanced on-shelf transport of deep waters, primarily in the bottom boundary layer, where the shelf narrows downwave, but only during upwelling. Thus certain places on the shelf will be locations of enhanced cross-shelf transport of cold, nutrient laden water, setting the stage for enhanced primary productivity which will occur when these waters are brought into the euphotic zone.

These four conclusions can alter theoretical and numerical models of the coastal ocean. Many analyses of data assume a simple, almost two-dimensional, picture of the coastal ocean, in which surface and bottom Ekman fluxes balance, only to find that this does not explain the observations well, especially on the mid- and outer-shelves. But as described above, the along-isobath flow, and thus the cross-isobath bottom Ekman transport, are forced by the winds and modified by changes in shelf width  $L^{\text{fric}}$  downwave. Near the coast,  $L^{\text{fric}}$  is small, so the flow is nearly locally forced. Farther offshore, this is not true. On an east coast mid-shelf, where the depth is 75m,  $r$  and  $f$  are as above, and the bottom slope  $10^{-3}$ ,  $L^{\text{fric}}$  is 560 km, a distance over which shelf width, bottom slope, and coastal windstress will usually have changed significantly. Thus the magnitude of the surface and bottom Ekman transports at a point will often be unequal.

Similar issues are important to numerical modelers of continental shelf circulation.  $L^{\text{fric}}$ , especially on wide shelves or near the shelf-break, will often be comparable or larger than the model domain. The outer- and mid-shelves of these models will be controlled by winds and bathymetry outside of the model domain, and thus by whatever open boundary condition the modeler sets on the upwave boundary. It will often be impractical, and always wasteful, to remove this problem by extending the model domain  $L^{\text{fric}}$  upwave, where  $L^{\text{fric}}$  is defined at the greatest depth of interest. Since the dynamics described above are barotropic, it would often be more practical to obtain the upwave surface pressure gradient boundary condition from a much more economical shallow water model whose domain extends  $L^{\text{fric}}$  upwave of the region of interest. The upwave boundary of the shallow water model could be forced by a coastal trapped wave model to capture even more remotely forced dynamics [*Battisti and Hickey, 1984*].

Further extension of this work will most likely describe the influence of stratification on the results described above. Cross-isobath advection of density in the bottom boundary layer can, at long time, cause bottom Ekman transport to shut down or be reduced [*Trowbridge and Lentz, 1991*]. This will eliminate or reduce the dominant adjustment mechanism to changed shelf width, cross-shelf bottom Ekman transport, and some other adjustment mechanism must become dominant. What this might be is, at present, unclear.

## **Acknowledgments**

I wish to thank Peter Franks for allowing me to pursue this idea, even as it drifted farther away from the bio-physical modeling I was to be doing. Ken Brink kindly reviewed the manuscript, and his suggestions, along with those of two anonymous reviewers, improved it greatly. This work was funded through grants NA76GP0176 and NA96OP0366 to Peter J.S. Franks from NOAA for North Atlantic GLOBEC program. This is GLOBEC contribution number XXX.

## Appendix: The numerical solution of (7)

The solution to the vorticity equation (7) described above are meant to be identical to the solution on a semi-infinite  $f$ -plane coastal ocean, but must be computed on a finite numerical domain. A coastal boundary condition, and three open boundary conditions, on the upwave, downwave, and offshore boundaries, must thus be specified. The coastal boundary condition is that there be no flow through the coast, so  $\Psi = 0$  at  $x = 0$ .

The most important of the open boundary conditions is that on the upwave cross-shelf boundary, for it is the “initial-condition” which the heat equation like (9) propagates through the domain. Since the domain upwave of the numerical domain is assumed to be without alongshore variation, a natural choice for boundary conditions is  $\Psi_y = 0$  at the upwave boundary.

The downwave boundary is harder to specify, but if the domain extends farther than  $L^{\text{fric}}$  downwave, the flow will have adjusted to the new shelf width and ceased to vary alongshore, so  $\Psi_y = 0$  at the downwave boundary would be appropriate. However, nearly any other boundary condition will work as well, for any boundary condition induced error is trapped near the boundary by the downwave propagation of information. For example, if there is a wall ( $\Psi = 0$ ) at the downwave boundary, any error is contained within a frictional boundary layer near the wall. This boundary layer is dynamically equivalent to the *Stommel* [1948] solution for the width of the Gulf Stream, and has a width  $\sqrt{r\beta^{-1}}$ , where  $\beta$  is the quasigeostrophic topographic potential vorticity gradient  $\beta$ . This width is only several hundred meters for the problem described above. Outside of this boundary layer, the flow is unaffected by the boundary condition. This has been verified numerically.

The offshore boundary condition should allow flow into the domain to adjust for the change in transport along the shelf, and thus  $\Psi$  ought to be allowed to vary along the boundary. If there is no wind at the boundary, there should be no alongshore flow (at least where the shelf width is uniform) at the boundary. Thus well offshore of the region of interest, the wind is allowed to

decay to zero, and  $\Psi_x$  is set to zero at the offshore boundary. However, the solution on the shelf is nearly unaffected by this boundary condition, because of the strong potential vorticity gradients on the shelf slope. This was verified by varying the offshore extent of the model and altering the offshore boundary condition.

The geometry of the coastal ocean shown in figures 1 to 5 was chosen so that the gradient of water depth was everywhere continuous. The shelf slopes linearly from depth  $h_0$  to depth  $h_0 + h_{sb}$  over a distance  $x_{sh}$ . Over the shelf-slope, the depth smoothly but rapidly increases to a depth  $h_{ab}$  over a distance  $x_{sh}$ , and offshore of  $-(x_{sh} + x_{sl})$  the bottom is flat at a depth of  $h_{ab}$ . The width of the shelf changes smoothly from  $x_{sh} = x_{upwave}$  to  $x_{sh} = x_{downwave}$  over a distance  $\Delta y$  centered around  $y = 0$ . For the results shown in figures 1 through 5,  $\Delta y = 25$  km, and the other parameters are as described in section 3.

Equation (7) is then solved with the mud2 elliptic equation solver, described in *Adams* [1989]. This is computationally inefficient, for it is solving a nearly hyperbolic system with an elliptic equation solver, but it does work.

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## Figure Captions

**Figure 1:** (Top) The flow forced by a uniform upwelling-favorable wind along a northern hemisphere shelf which narrows to the north, found by numerically solving (7). (Bottom) The flow forced by a uniform downwelling-favorable wind along a northern hemisphere shelf which widens to the north. For both the top and bottom panels: The heavy line marks the 110 meter isobath, which is the depth of the shelf-break. A) Depth averaged velocity. B) Along-isobath velocity scaled by the along-isobath velocity which would exist in the absence of any change in shelf width, from equation (2). Shaded where solution deviates from two dimensional solution by more than 40%. C) Net cross-isobath transport across the indicated isobath. Along- and cross-shelf axes not to scale, and arrow lengths in panel A) and C) not equivalent, for one is a transport, and the other a velocity.

**Figure 2:** The new cross- and along-isobath coordinate system. The thin lines are the isobaths. The coordinate  $p$  is parallel to isobaths, and  $n$  is normal.  $p$  takes the place of  $y$  and  $n$  takes the place of  $x$  if the isobaths parallel the coast.  $R$  is the radius of curvature of the isobaths.

**Figure 3:** Along-isobath velocity scaled by the along-isobath velocity which would exist in the absence of any change in shelf width, as in panel B of figure 1. Marked on the plot is the cross-shelf position of the point of maximum along-isobath velocity (thick solid line), and the locus where  $L^{\text{fric}}$  on an isobath matches the distance downwave from the change in shelf width (dashed line).

**Figure 4:** Left: the bottom Ekman cross-isobath transport. Right: the geostrophic cross-isobath transport, both normalized by the strength of the surface Ekman transport, and both for the upwelling wind case shown in figure 1. The heavy offshore line marks the shelf-break, the lighter

line the 75 meter isobath. Along- and cross-shore axes not to scale. Bottom Ekman transport calculated from the bottom stress obtained in the numerical solution to (7), and the geostrophic transport from along-isobath pressure gradients computed from the same solution.

**Figure 5:** Top: total cross-isobath transport from numerical solution to (7) and from estimate in (13), both scaled by surface Ekman transport. Bottom: cross-isobath surface Ekman, bottom Ekman, and geostrophic transports from numerical solution to (7) and from estimate in (13), also scaled by the surface Ekman transport. Both are calculated along the 75 meter isobath marked in figure 4.  $L^{\text{fric}}$  is 84 km on the 75 m isobath. As the flow adjusts to the new shelf width, the cross-shelf transport relaxes back to the two dimensional solution.

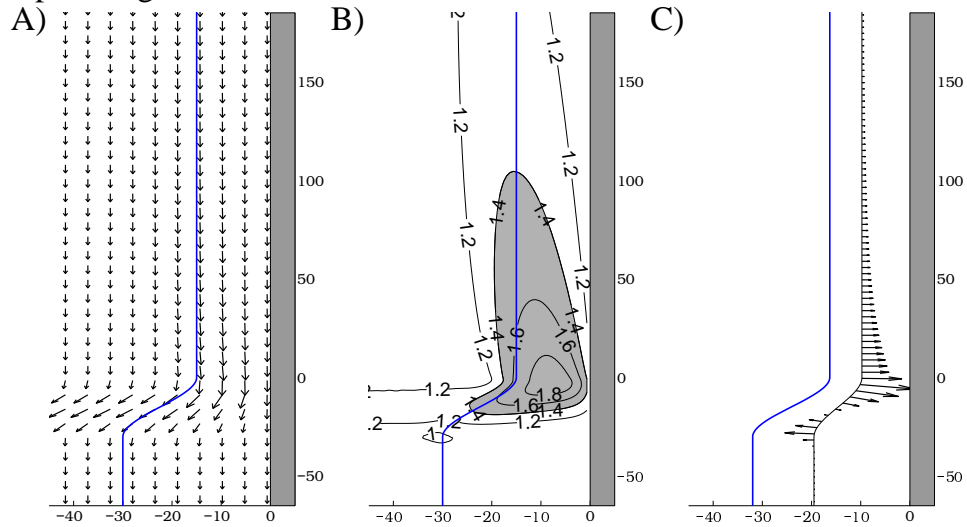
**Figure 6:** Additional water is forced onto the shelf when there is upwelling and the shelf narrows downwave, or when there is downwelling and the shelf widens downwave. Both are illustrated above. Only the former forces deep, cold, nutrient laden waters onto the shelf, because it is only when the alongshore flow is accelerated that the bottom Ekman transport ( $U_{Ek}^{\text{bot}}$ ) is enhanced over the surface Ekman transport ( $U_{Ek}^{\text{top}}$ ). Note: alongshore velocity ( $v$ ) arrows and cross-shelf transport arrows have different scale.

**Figure 7:** An idealized model of the region north of Monterey Bay and south of Cape Mendocino. The contours are of the along-isobath velocity, normalized by the velocity along a shelf whose width is constant. The thick black line marks the shelf break. MR, S.F., Pt.R, and Pt.A mark the positions of Monterey, San Francisco, Pt. Reyes and Pt. Arena, respectively. The map is not to scale: cross-shelf distance is exaggerated relative to alongshelf distance.

**Figure 8:** The coast of Northern America from Mexico to the Northern California border. The two-hundred meter isobath is included, and regions where the shelf narrows downwave for more

than a hundred kilometers are labeled.

### Upwelling



### Downwelling

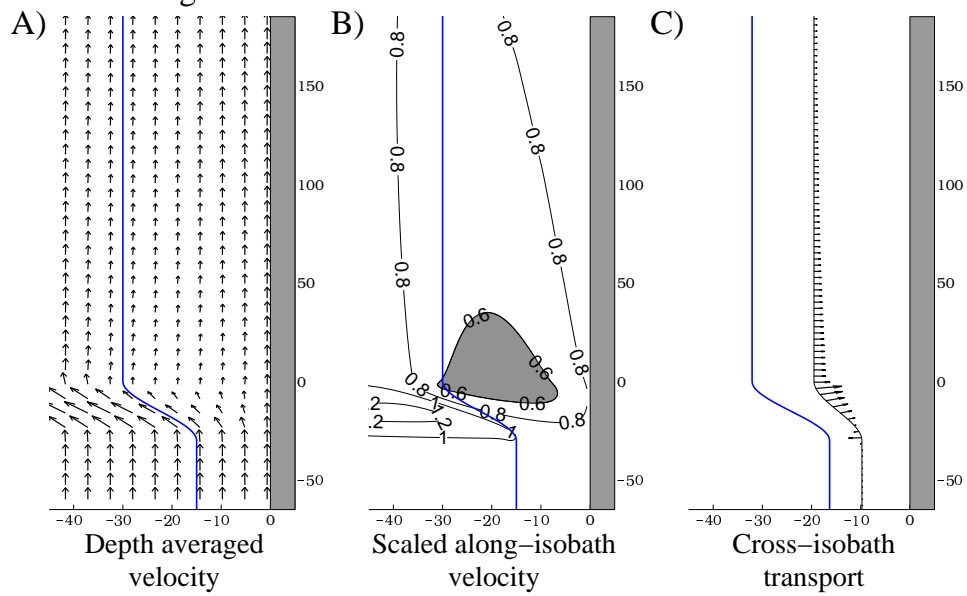


Figure 1:

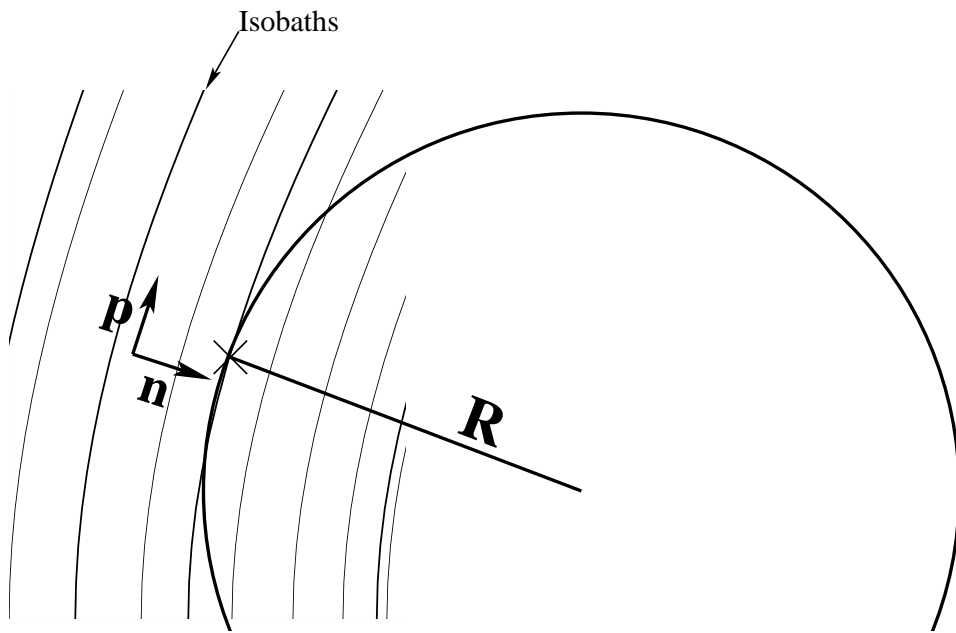


Figure 2:

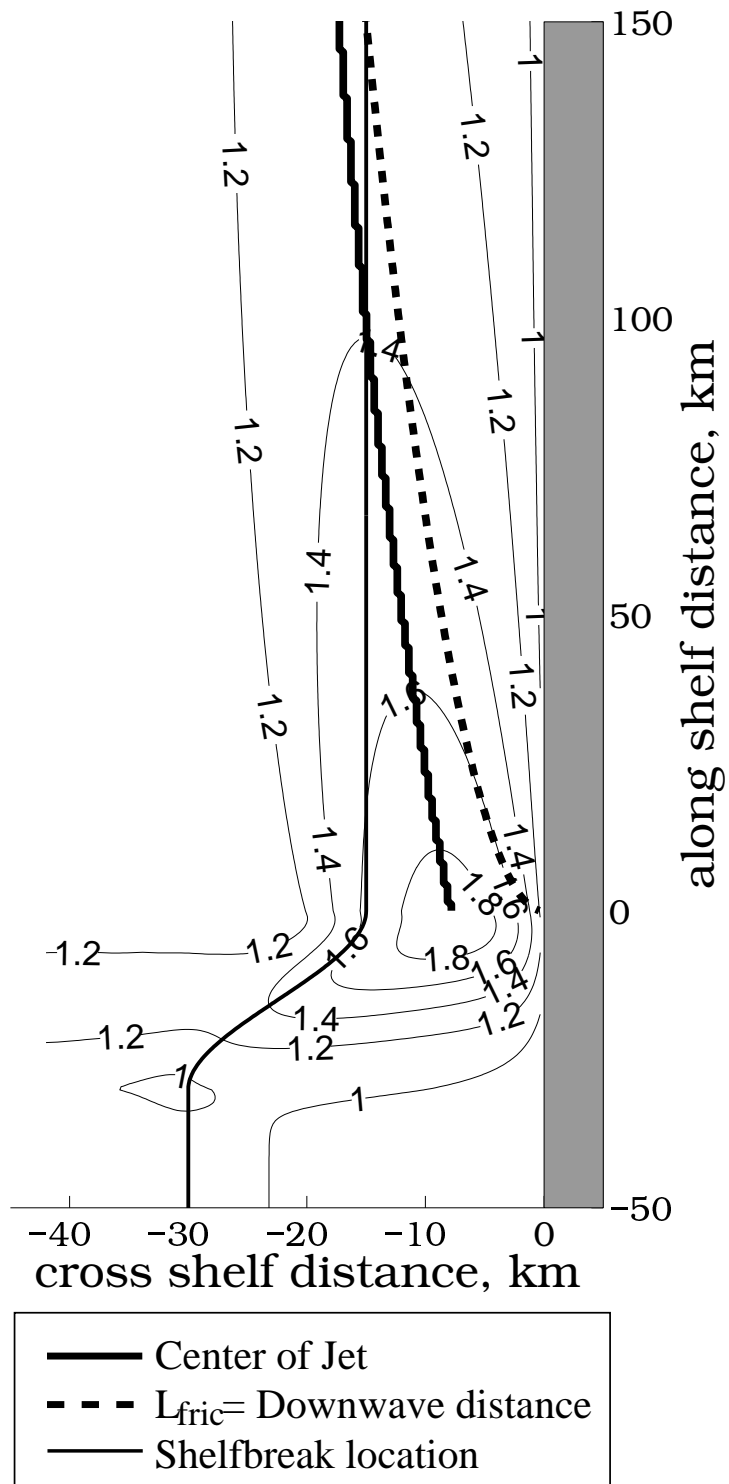


Figure 3:

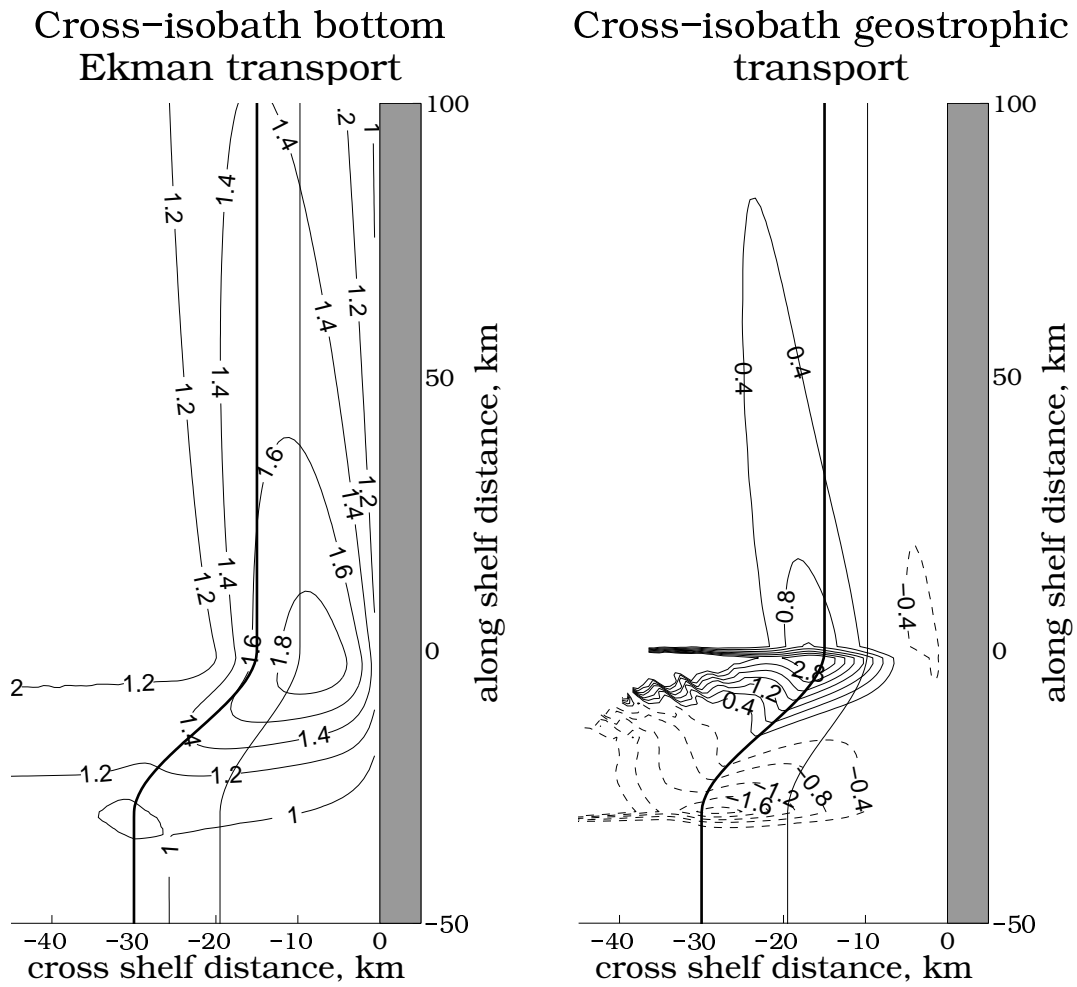


Figure 4:

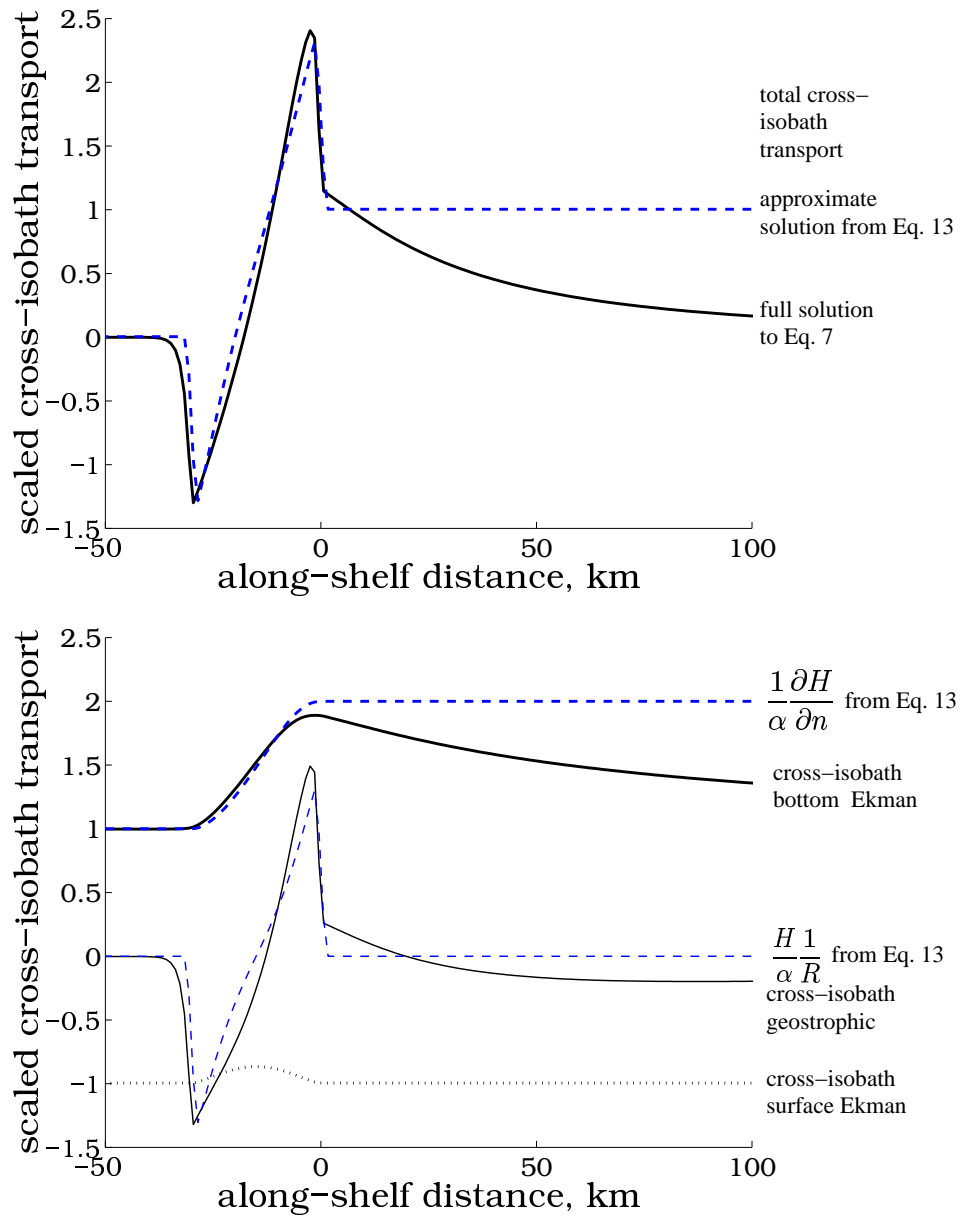
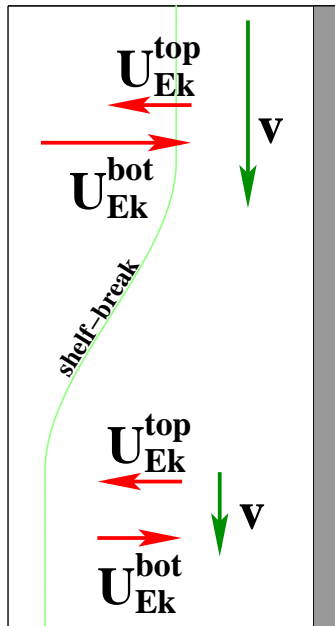


Figure 5:



A) **Upwelling**  
net onshore transport  
of deep waters



B) **Downwelling**  
net onshore transport  
of surface waters

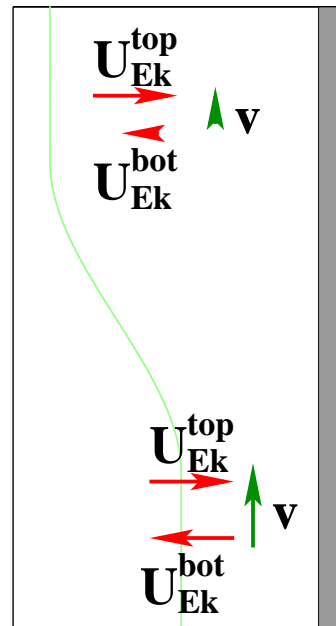


Figure 6:

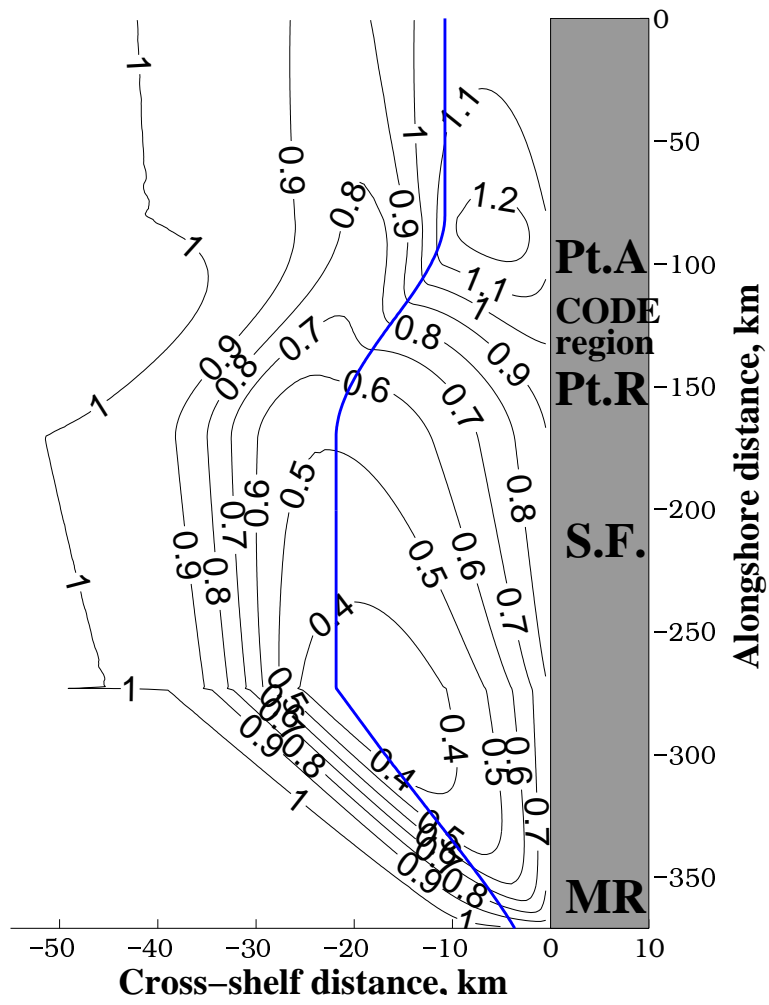


Figure 7:

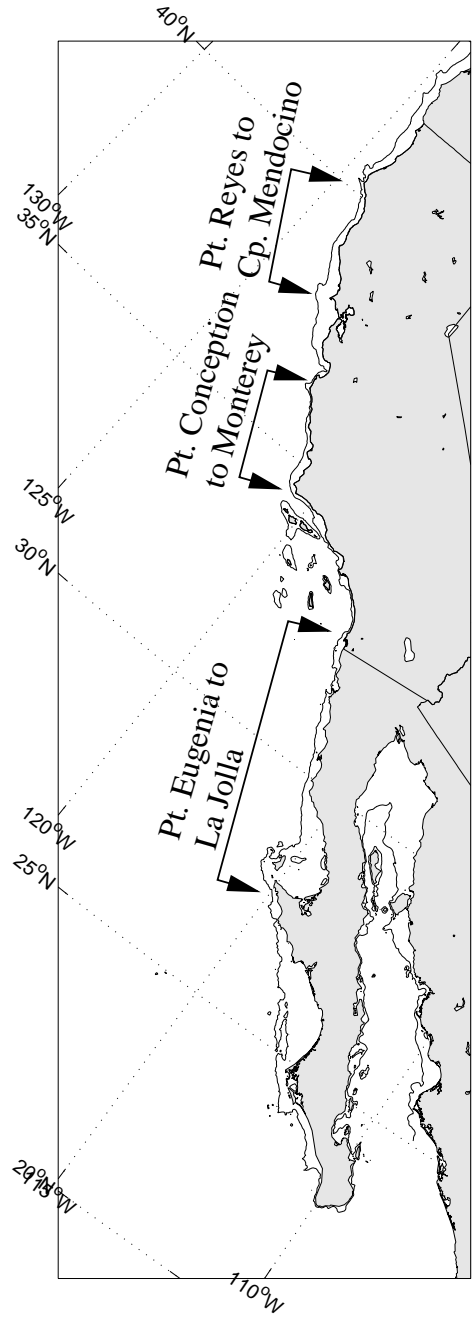


Figure 8: